B	is the gap perimeter;
η	is the viscosity;
z	is the vertical coordinate;
$H_0 = \alpha_0 k\rho^2 g\beta(\Delta T)^2 B\delta/12\eta\overline{T}$; $K = g^2k^2\rho^2\beta^2(\Delta T)^2 \cdot B\delta^3/120\eta^2 D_{eff}$;	
k	is the permeability.

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RADIATIVE ANGULAR COEFFICIENTS IN AXISYMMETRIC SYSTEM

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UDC 536.3

A method is proposed for the calculation of the angular coefficients from an analytic determination of the region of visibility.

The design of furnaces, high-temperature chemical equipment, high-temperature energy-conversion apparatus, and cryogenic systems involves calculations of radiative heat transfer.

Because of the complexity of radiative heat transfer and the lack of accurate values of the emissive characteristics of surfaces, it is usual in calculations to consider models and shells with simple surface properties.

In calculating the radiative heat transfer between diffusely emitting and diffusely reflecting surfaces separated by a diathermal medium, it is necessary to determine the angular coefficients of the radiation, which determines the proportion of the energy transfer transmitted from one surface to another.

There are a considerable number of works in which the angular coefficients are calculated analytically for various configurations ([1-4], etc.). The present paper proposes an algorithm for the computer calculation of radiative angular coefficients.

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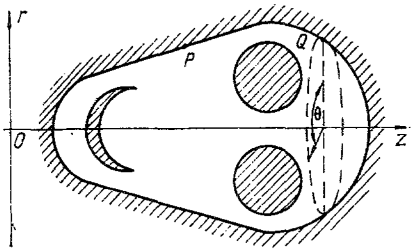

Fig. 1. Axisymmetric system: z is the coordinate along the axis of rotation; r is the distance from the axis; θ is the angle of rotation about the axis.

In the case when the radiative flux density is uniformly distributed over the surface, the angular coefficient between the surfaces p and q is

$$\varphi_{pq} = \frac{\int\int_{A_p A_q} \frac{\cos\beta_i \cdot \cos\beta_j}{\pi L_{ij}^2} dA_i dA_j}{\int_{A_p} dA_i}, \tag{1}$$

where A_p and A_q are the surface areas of p and q; L_{ij} is the distance between the elements i and j of the surfaces; β_i and β_j are the angles between the internal normals to the surface elements and the lines containing these elements.

The region of integration extends over those parts of the surfaces p and q which are directly visible to one another.

The angular coefficients are calculated in an axisymmetric system consisting of a cavity with interior bodies (Fig. 1). The regions p and q adopted in this case are bands of the surface r = r(z) cut by the planes z = const. Each region p(q) is divided into smaller bands i(j). The required integral is calculated for each pair i, j and then for each pair p, q these integrals are summed over all the i bands into which the region p is divided and over all the j bands into which region q is divided.

The integral is calculated using the following formula:

$$\frac{1}{A_i} \int\int_{A_i A_j} f_{ij} dA_i dA_j = \frac{A_j}{2\pi^2} \int_0^{\theta_{ij}} f_{ij}(r_i, z_i, 0, r_j, z_j, \theta)\, d\theta. \tag{2}$$

The following theorem is valid: In an axisymmetric configuration, the region of visibility of the points $p(r_i, 0, z_i)$ and $Q(r_j, \theta, z_j)$ determined by an arbitrary solid of revolution consists of not more than two arcs in $[0, \pi]$.

Consider the interfering body formed by rotation of the segment $r_I r_F$ about the z axis (Fig. 2). Three cases are possible:

a) r_I and r_F are any finite values, $r \neq 0$;

b) $r_F = 0$, $r_I > r_F$, $r_I = \text{const}$;

c) $r_I \to \infty$, $r_F \neq 0$.

Now construct two cones with vertex at p and directrices passing through the points $r = r_I$, $z = z_\lambda$ (cone I) and $r = r_F$, $z = z_\lambda$ (cone II) (case a). When the point Q is rotated about the axis, there is visibility between the points P and Q when the ray PQ lies outside cone I or inside cone II. Hence, the region of visibility of points P and Q for $\theta \in [0, \pi]$ consists of no more than two arcs: $[0, \theta_{ij}^F]$, $[\theta_{ij}^I, \pi]$.

For $r_F = 0$ (case b) the region of visibility consists of one arc $[0, \theta_{ij}^F]$; for $r_I \to \infty$ (case c) the region of visibility is $[\theta_{ij}^I, \pi]$.

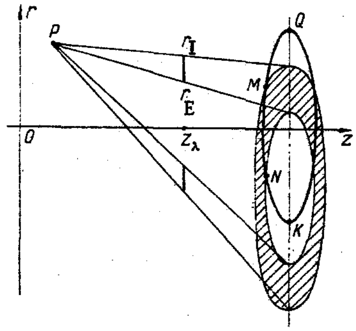

Fig. 2. Region of visibility of points P and Q: $\breve{MQ}$, $\breve{NK}$

An interesting body lying on the axis (Fig. 3a) may be represented as a set of large number of sufficiently thin cylinders (case b), for which $r_{F\lambda} = 0$, $r_{I\lambda} = \mathrm{const}(\lambda)$, where λ is the cylinder number. The region of visibility Δ_{ij} determined by the given interfering body is

$$\Delta_{ij} = [0,\ \theta^F_{ij}],$$

where

$$\theta^F_{ij} = \min_\lambda \theta^F_{ij}{}_\lambda .$$

Analogously, the region of visibility determined by the surface of an axisymmetric cavity (Fig. 3b) is $[\theta^I_{ij},\ \pi]$, where

$$\theta_{ij} = \max_\lambda \theta^I_{ij}{}_\lambda .$$

The region of visibility determined by an interfering body of toroidal type (Fig. 3c) is in the general case

$$\Delta_{ij} = [0,\ \theta^F_{ij}] \cap [\theta^I_{ij},\ \pi],$$

where

$$\theta^F_{ij} = \min_\lambda \theta^F_{ij}{}_\lambda,\quad \theta^I_{ij} = \max_\lambda \theta^I_{ij}{}_\lambda .$$

Consider a system for which the interfering bodies are a body lying on the axis and the surface of an axisymmetric cavity. The region of visibility for any pair of points P and Q consists of one arc $[\theta^I_{ij},\ \theta^F_{ij}]$.

The necessary and sufficient conditions for there to be visibility between the points $P(r_i, 0, z_i)$, $Q(r_j, \theta, z_j)$ are: 1) that the angle β_i and β_j be acute; 2) that the segment PQ encounter no interfering bodies.

Assume that the surfaces of the interfering bodies are either cylinders, cones, or spheres. These surfaces may be described by the general equation

$$r^2_\lambda = k_\lambda (z - \xi_\lambda)^2 + R^2_\lambda, \tag{3}$$

where λ is the number of the surface; ξ_λ is the coordinate of the sphere center or the vertex of the cone; $k_\lambda = -1$ for a sphere; $k_\lambda = \tan^2\psi$ for a cone ($\psi + \pi/2$ is the angle between the internal normal to the surface and the positive direction of the axis of rotation); $k_\lambda = 0$ for a cylinder; $R_\lambda = 0$ for a cone; R_λ is the radius in the case of a sphere or a cylinder.

The straight line PQ and the interfering surface in Eq. (3) determine the coordinate z of the point of intersection

$$az^2 - 2bz + c = 0, \tag{4}$$

where

$$a = r_i^2 + r_j^2 - 2r_ir_j\cos\theta - k_\lambda (z_j - z_i)^2 = m + h\cos\theta;$$
$$b = r_i^2 z_j + r_j^2 z_i - r_ir_j (z_i + z_j)\cos\theta - k_\lambda\xi_\lambda (z_j - z_i)^2 = l + t\cos\theta;$$
$$c = r_i^2 z_j^2 + r_j^2 z_i^2 - 2r_ir_jz_iz_j\cos\theta - (k_\lambda\xi^2_\lambda + R^2_\lambda)(z_j - z_i)^2 = s + g\cos\theta.$$

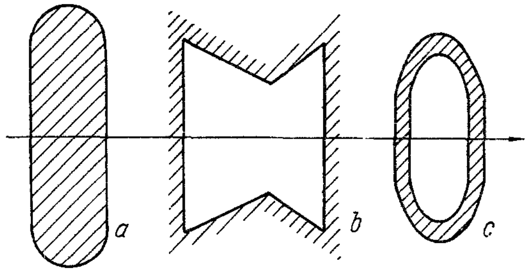

Fig. 3. Different types of interfering body.

The ray PQ intersects the interfering body if the roots of Eq. (4) are real and the point of intersection (r_p, z_p) lies:

1) between P and Q

$$z_i \leqslant z_p \leqslant z_j, \tag{5}$$

2) on the surface of the interfering body but not on its continuation (the interfering body is bounded on the z axis by the coordinates $z_{I\lambda}$, $z_{F\lambda}$)

$$z_{I\lambda} \leqslant z_p \leqslant z_{F\lambda}. \tag{6}$$

To determine the region of visibility, those among the interfering bodies which affect the angle of visibility of the points P and Q are selected, i.e., those for which

$$z_i \leqslant (z_{I\lambda} \vee z_{F\lambda}) \leqslant z_j. \tag{7}$$

Then the interval of visibility is determined independently for each interfering surface.

The procedure for determining the region of visibility of the points P and Q ($z_i \neq z_j$) for one interfering surface is as follows:

1) to determine the visibility between the points P and Q for $\theta = 0$,

2) to verify that point Q is visible from point P for $\theta = \pi$.

As a result of steps 1 and 2 for the interfering surface, one of four possibilities may be found:

	I	II	III	IV
Visibility at $\theta=0$	yes	no	yes	no
Visibility at $\theta=\pi$	yes	no	no	yes

The region of visibility of points P and Q is $[0, \pi]$ in case I, zero in case II, $[0, \theta_\lambda^F]$ in case III, and $[\theta_\lambda^I, \pi]$ in case IV (θ_λ^F is the angle at which the straight line cuts the interfering surface when the point Q is rotated about the z axis).

The equation of the tangent to the interfering surface gives

$$\cos\theta_\lambda^F = \begin{cases} \dfrac{R_\lambda^2 - \xi_\lambda^2 - z_i z_j + \xi_\lambda (z_i + z_j) - \sqrt{(r_i^2 - \rho_i^2)(r_j^2 - \rho_j^2)}}{r_i r_j} & \text{for a sphere} \\ \dfrac{\rho_i \rho_j - \sqrt{(r_i^2 - \rho_i^2)(r_j^2 - \rho_j^2)}}{r_i r_j} & \text{for a cone or cylinder} \end{cases} \tag{8}$$

$$\rho_v^2 = k_\lambda (z_v - \xi_\lambda)^2 + R_\lambda^2, \quad v = i,\ j.$$

The tangent determines the angle of visibility if Eqs. (5) and (6) hold for the coordinates of the point of contact.

If there is no tangent, the angle at which the straight line passes through the edge of the interfering surface ($z_{I\lambda}$ or $z_{F\lambda}$) is determined

$$\cos\theta_\lambda^F = \frac{2lz_{I\lambda} - s - mz_{I\lambda}^2}{hz_{I\lambda}^2 - 2tz_{I\lambda} + g}. \tag{9}$$

TABLE 1. Angular Coefficients φ_{ij}

Region No.	1	2	3	4	5	6	7	8
3	0	0	0	0	0	0,06688	0,27174	0,1512
30	0	0	0	0,00061	0,0009	0	0	0,00051
34	0	0	0	0	0	0	0	0

Region No.	9	10	11	12	13	14	15
3	0,10993	0,08134	0,06119	0,00044	0,01463	0,01596	0,01578
30	0,00086	0,00155	0,00306	0,04347	0,04628	0,10031	0,19174
34	0,02097	0,10796	0,16726	0,06494	0,11469	0,09496	0,07791

Region No.	16	17	18	19	20	21
3	0,01369	0,01121	0,00885	0,00712	0,00583	0,01827
30	0,24544	0,18213	0,09302	0,04284	0,02047	0,02053
34	0,06153	0,03957	0,01982	0,00961	0,00498	0,00417

Region No.	22	23	24	25	26	27	28	29	30
3	0,00849	0	0	0	0	0	0	0	0
30	0,0066	0	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0	0	0

Region No.	31	32	33	34	35
3	0	0,02292	0	0	0,11609
30	0	0	0	0	0
34	0	0,14709	0,06548	0	0

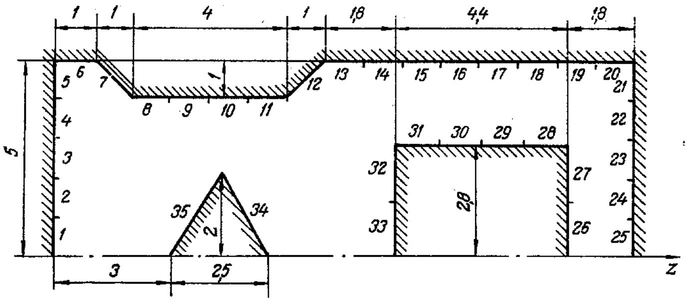

Fig. 4. Configuration of system whose angular coefficients are shown in Table 1; z is the axis of rotation. The division into regions is shown. The dimensions are given in cm.

If the straight line PQ passes through both ends of the interfering surface when the point Q is rotated about the z axis, the smallest of the two resulting values of θ^F_λ is chosen (θ^I_λ is determined from the condition that the straight line PQ passes through the edge of the interfering surface).

Considering the set of interfering surfaces in Eq. (3), the following sequences are obtained:

$$\mathrm{I}: \theta^I_1,\ \theta^I_2,\ \ldots,\ \theta^I_n;$$
$$\mathrm{II}: \theta^F_1,\ \theta^F_2,\ \ldots,\ \theta^F_n,$$

where 1, 2, ..., n is the number of the interfering surface.

The region of visibility for points P and Q is $(\theta^I_{ij}, \theta^F_{ij})$, where

$$\theta^I_{ij} = \max_\lambda \theta^I_\lambda;\quad \theta^F_{ij} = \min_\lambda \theta^F_\lambda.$$

If $\theta^I_{ij} > \theta^F_{ij}$, the point Q is not visible from P.

For the case $z_i = z_j$, the only bodies that may interfere are those whose projections on the z axis include the projections of the points P and Q.

If there is no visibility between P and Q for $\theta = 0$, the region of visibility is 0. If there is visibility for $\theta = 0$ and $\theta = \pi$, the region of visibility is $[0, \pi]$. If there is zero visibility for $\theta = \pi$, the angle of visibility is determined from the condition of tangency of the straight line PQ at the interfering surface

$$\cos\theta_\lambda^F = \frac{\rho_\lambda^2 - \sqrt{(r_i^2 - \rho_\lambda^2)(r_j^2 - \rho_\lambda^2)}}{r_i r_j}. \tag{9'}$$

The integrand in Eq. (2) is

$$f_{ij} = \frac{1}{\pi}\,\frac{(A_{ij} + B_{ij}\cos\theta)(C_{ij} + D_{ij}\cos\theta)}{(E_{ij} + F_{ij}\cos\theta)^2},$$

and the coefficients A_{ij}, B_{ij}, C_{ij}, D_{ij}, E_{ij}, F_{ij} are calculated from the values of r_i, z_i, ψ_i, r_j, z_j, ψ_j.

The integral $I = \int\limits_{\theta_{ij}^I}^{\theta_{ij}^F} f_{ij}\,d\theta$ is calculated from the trapezium rule.

For small angles ($\theta \leq 30°$) and for $z_i = z_j$, an analytic expression may be obtained for the integral.

As an illustration, Table 1 gives the angular coefficients calculated for the system shown in Fig. 4.

The system includes 166 computational points. Regions 1-6, 8-11, 13-25, and 28-31 are divided into two computational points, regions 7, 12, 26, 27, 32, and 33 are divided into three, and regions 34 and 35 into five; the division is uniform over the length.

As is evident from Table 1, the accuracy of the angular coefficients obtained using angular-coefficient algebra is sufficiently high: $\sum\limits_{j=1}^{35} \varphi_{3,j} = 1.0016$, $\sum\limits_{j=1}^{35} \varphi_{30,j} \approx 1.00032$, $\sum\limits_{j=1}^{35} \varphi_{34,j} = 1.000999$. At the corners (regions 5-6 and 20-21) the accuracy of the calculation is rather lower.

The time required for the calculation on a BESM-6 computer was 20 sec.

In determining the region of visibility by the traditional method (the region $[0, \pi]$ is divided into 50 intervals of equal length and the visibility is determined for each interval) the same accuracy requires 10-20 times the calculation time.

NOTATION

p, q, i, j	are the surface numbers;
A_p	is the surface area of p;
φ_{pq}	is the angular coefficient between surfaces p and q;
r, z, θ	are the cylindrical coordinates;
β_i	is the angle between internal normal to surface i and the line joining surfaces i and j;
L_{ij}	is the distance between elementary areas dA_i, dA_j;
P	is the point of i-th surface;
Q	is the point of j-th surface;
λ	is the number of interfering surface;
k, ξ, R	are the parameters of interfering surface;
n	is the number of interfering surfaces;
Δ_{ij}	is the region of visibility of points P and Q;
$\psi + \pi/2$	is the angle between internal normal to surface and positive direction of axis of rotation.

Indices

I	is the initial (at beginning of interval);
F	is the final (at end of interval).

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REFLECTIVE POWER OF TWO-PHASE MEDIA OF CYLINDRICAL GEOMETRY

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The brightness of the radiation reflected from a cylinder filled with particles of known optical properties is considered. The dependence of the reflective power on the optical properties of the medium and the experimental conditions is investigated in the single-scattering approximation. The limits of applicability of the method are estimated.

In determining the reflective power of two-phase media of cylindrical geometry, the approximation most commonly used is that of Eddington (see [1, 2], for example). As shown in [3, 4], it may correctly be used to calculate the emissive characteristics of two-phase media of nonplanar geometry. However, when external radiation is incident on a finite two-phase medium, the use of the Eddington approximation requires particular caution, especially for media of optical thickness $\tau \leqslant 1$–3. In the present work, the single-scattering approximation is used to calculate the reflective power of such media and its dependence on the optical properties of the medium and the experimental conditions is analyzed.

The solution of the radiation-transfer equation in a two-phase medium may be written in the form (see [5], for example)

$$I(\mathbf{s},\ \mathbf{l}) = I(0,\ \mathbf{l}) \exp\left[-\int\limits_0^s \alpha(s')\,ds'\right] + \int\limits_0^s J(s')\ \exp\left[-\int\limits_{s'}^s \varkappa(s'')\,ds''\right] ds'. \tag{1}$$

Here $I(\mathbf{s}, \mathbf{l})$ is the radiation intensity at the point $\mathbf{s}$ in the direction $\mathbf{l} = \mathbf{l}(\theta, \varphi)$; $I(0, \mathbf{l})$ is the intensity of the external radiation; $J(\mathbf{s})$ is the emissive power of an elementary volume of the medium; $\alpha = \varkappa + \sigma$ is the attenuation coefficient, equal to the sum of the absorption and scattering coefficients.

Since $J(\mathbf{s})$ depends on $I(\mathbf{s}, \mathbf{l})$ in the scattering medium, Eq. (1) may only be solved by numerical methods. Limiting consideration to the case of single (nonmultiple) scattering, a solution of the problem may be obtained by replacing $J(\mathbf{s})$ in Eq. (1) by the distribution function for the sources created by the external radiation. In the case of nonplanar media, the integral term in Eq. (1) requires special consideration. Its physical meaning in the context of single scattering is fairly simple. It is the sum of the contributions of the radiation from each point of the medium in a given direction, taking into account attenuation.

Consider a medium of cylindrical geometry containing particles of known optical properties. The chosen coordinate system is shown in Fig. 1a: the x axis, from which η is measured is chosen in the plane containing the direction of the external radiation and the cylinder axis. The angles η and φ are positive when measured in the counterclockwise direction and negative in the opposite case. The angle θ, characterizing the direction of observation of the scattered radiation, is measured from the z axis. It is simple to show that, for normal incidence of the external radiation, the distribution of radiation sources in the cylindrical medium is given by the relation

$$S = S(r,\ \eta,\ \mathbf{l}) = \frac{1}{\alpha}\ J = \frac{\lambda}{4\pi}\ p(\gamma)\, I_0 \exp\left[-T(r,\ \eta)\right], \tag{2}$$

where I_0 is the external-radiation intensity in the direction $\mathbf{l}_0 = \mathbf{l}_0(\theta_0, \varphi_0)$; $p(\gamma)$ is the scattering index for an elementary volume; γ is the angle between the incident and observed radiation; $\lambda = \sigma/(\varkappa + \sigma)$ is the probability of survival of a quantum; and:

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